Coordinates System

Part 1

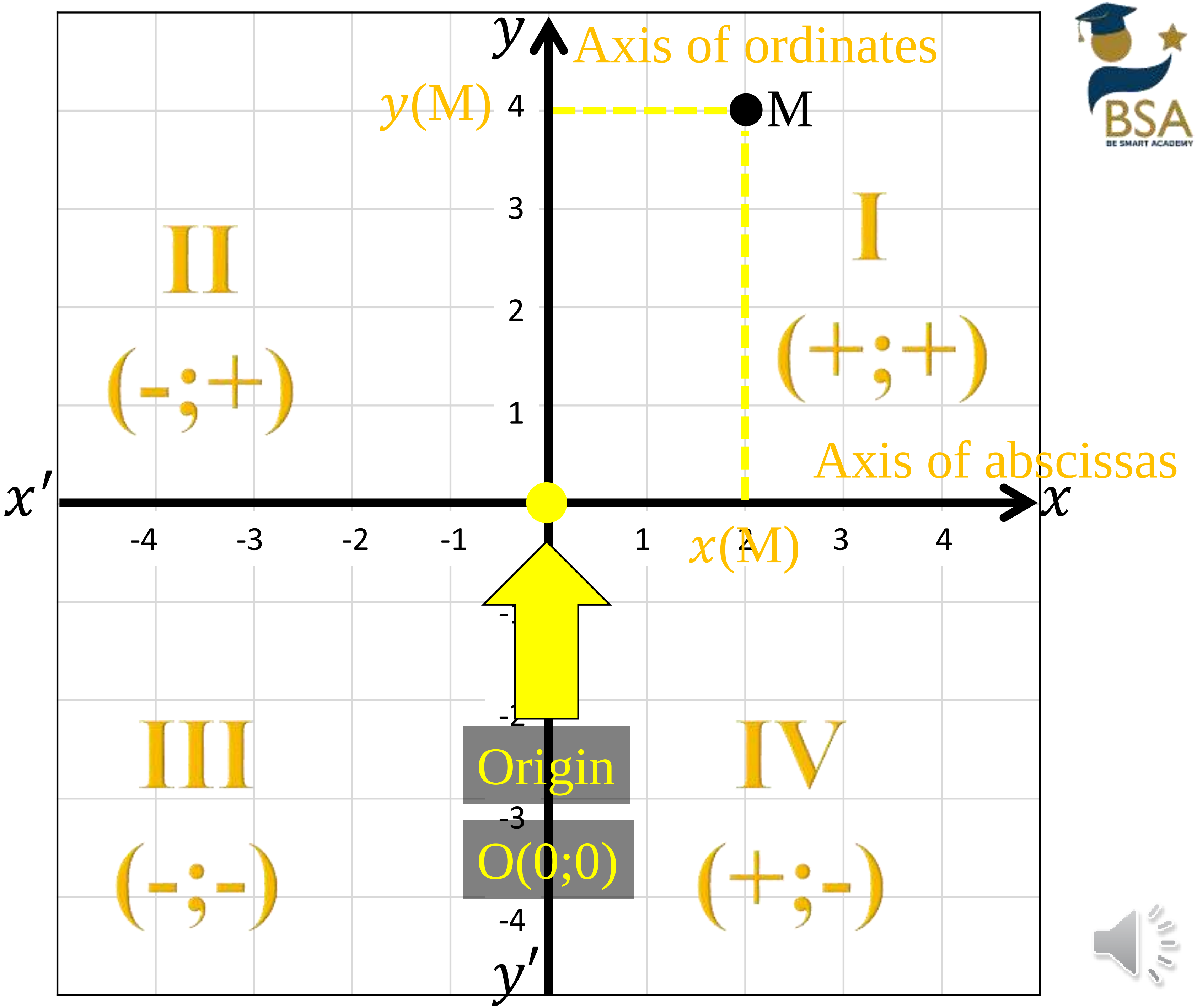
Recall

☞Coordinates system

ordinate

$M(x;y)$

abscissa



☞Length of a segment

$$AB=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}$$

Example:

A(-4;-2) and B(2;2)

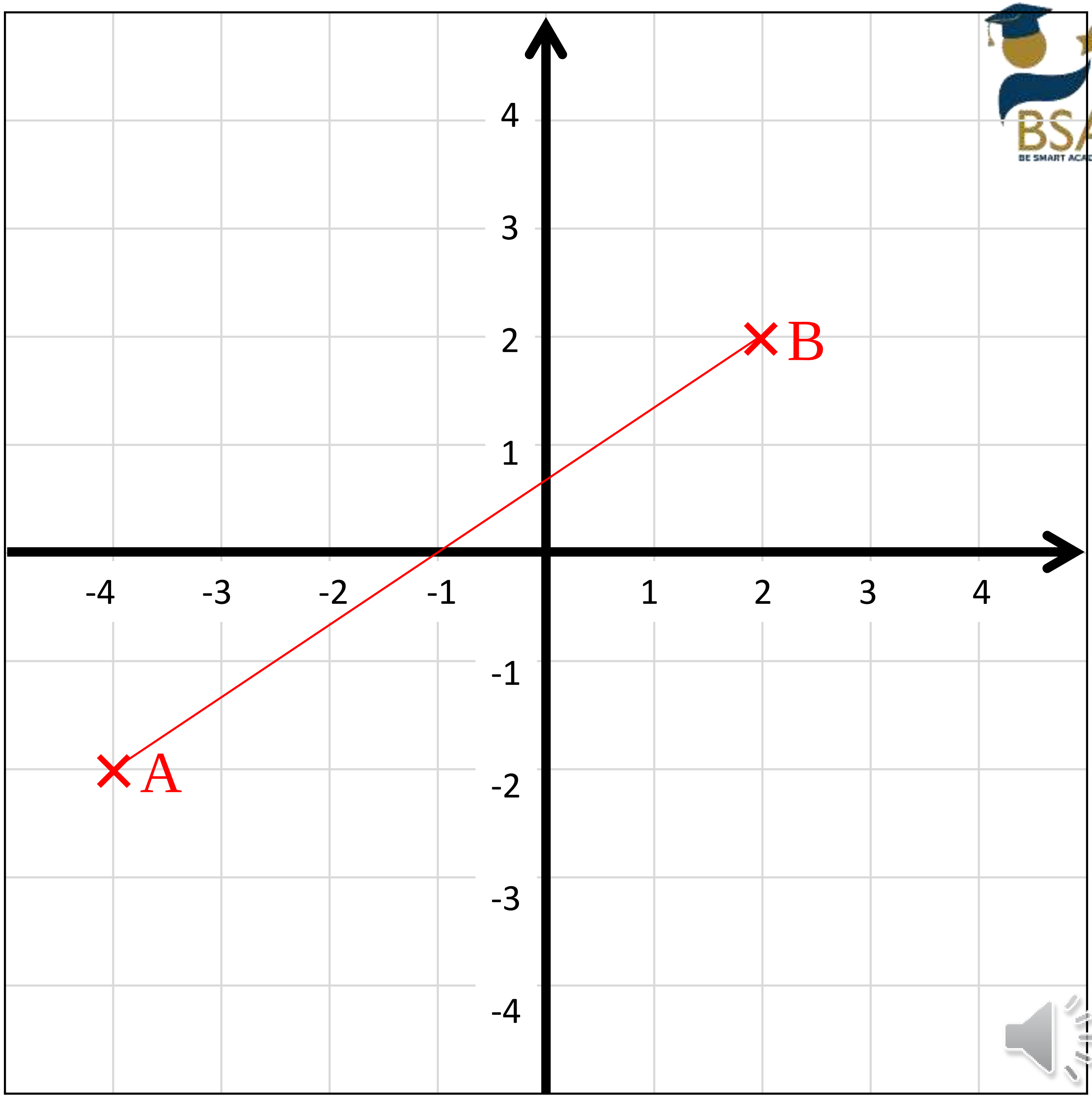
$$AB=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}$$

$$=\sqrt{\left(2-(-4)\right)^2+\left(2-(-2)\right)^2}$$

$$=\sqrt{36+16}$$

$$=\sqrt{52}$$

$$=2\sqrt{13}$$



☞Midpoint of a segment

I is the midpoint of [AB]:

$$x_I = \frac{x_A + x_B}{2}$$

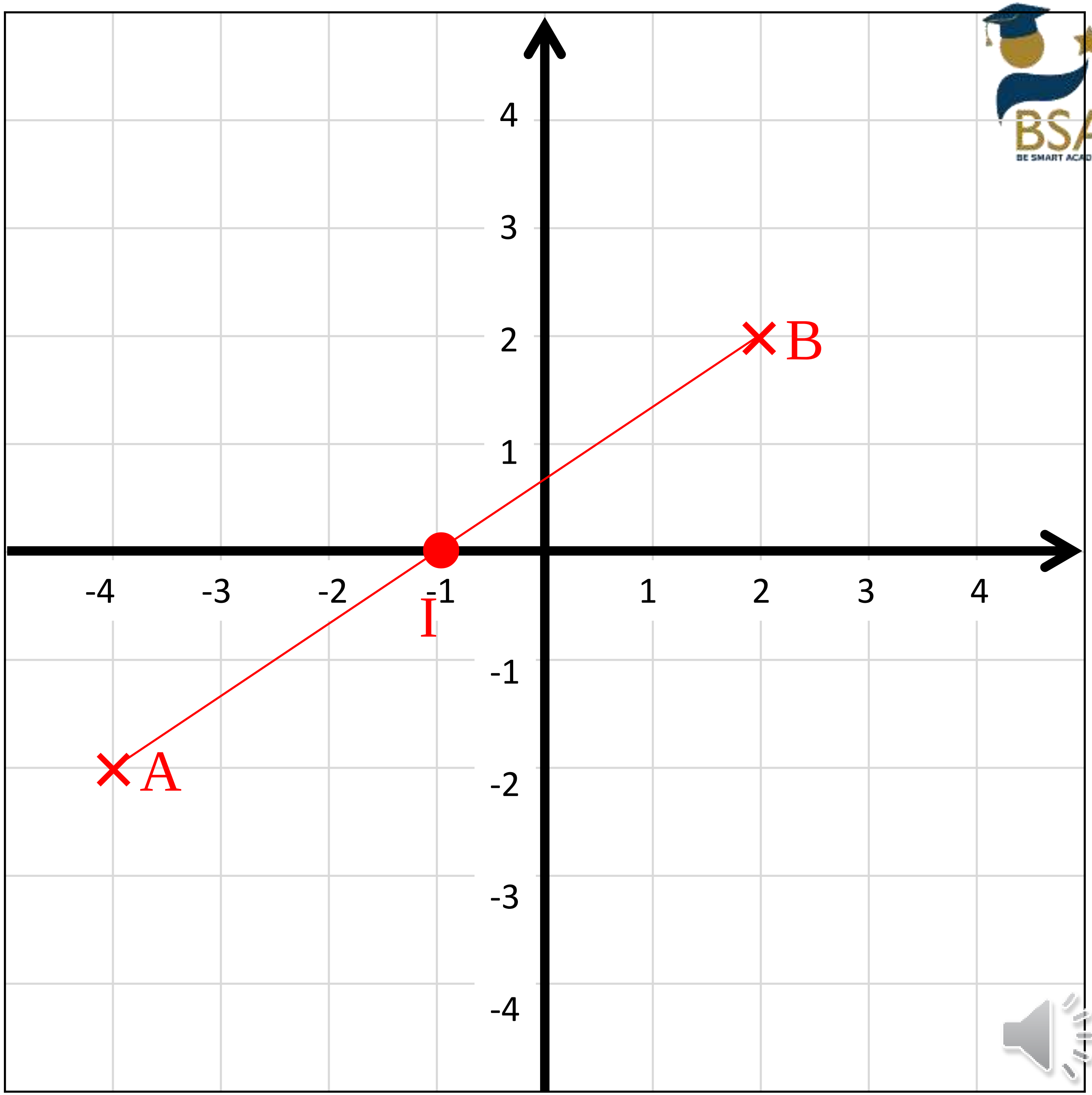
$$y_I = \frac{y_A + y_B}{2}$$

Example:

A(-4;-2) and B(2;2)

$$x_I = \frac{x_A+x_B}{2} = \frac{-4+2}{2} = -\frac{2}{2} = -1$$

$$y_I = \frac{y_A+y_B}{2} = \frac{-2+2}{2} = \frac{0}{2} = 0$$



☞ Application # 1

Consider the two points $A(-2; 5)$ and $B(6; -3)$.

1. Calculate the coordinates of:

   a. M the midpoint of [AB].

   b. N the symmetric of A with respect of B.

a. M is the midpoint of [AB], so:

$$x_M = \frac{x_A + x_B}{2} = \frac{-2+6}{2} = \frac{4}{2} = 2$$

$$y_M = \frac{y_A + y_B}{2} = \frac{5+(-3)}{2} = \frac{2}{2} = 1$$

So $M(2; 1)$

☞ Application # 1

Consider the two points $A(-2;5)$ and $B(6;-3)$.

1. Calculate the coordinates of:

   a. M the midpoint of [AB].

   b. N the symmetric of A with respect of B.

b. N is the symmetric of A with respect to B so:

B is the midpoint of [AN]

$$x_B = \frac{x_A + x_N}{2} \quad ; \quad y_B = \frac{y_A + y_N}{2}$$

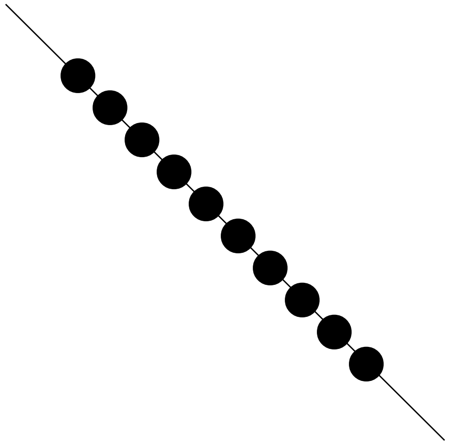
$$6 = \frac{-2 + x_N}{2} \qquad -3 = \frac{5 + y_N}{2}$$

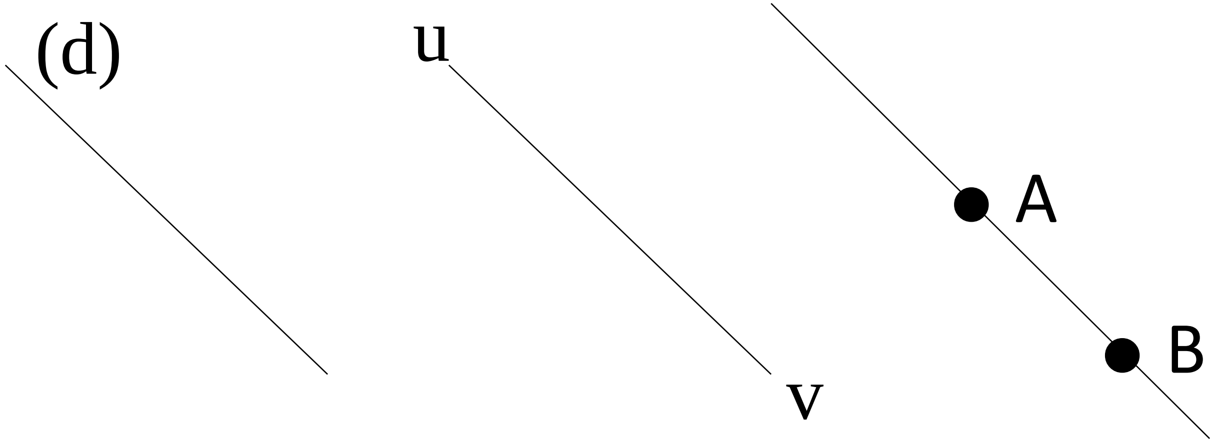
$$x_N = 2 \times 6 + 2 = 14 \qquad y_N = -3 \times 6 - 5 = -23$$

Then N (14;-23)

☞Line (Recall)

- A line is a set of points.
- A line is determined by two points.
- To name a line, we can use:
  - One small letters: (d)
  - Two small letters: (uv)
  - Two points of the line: (AB)





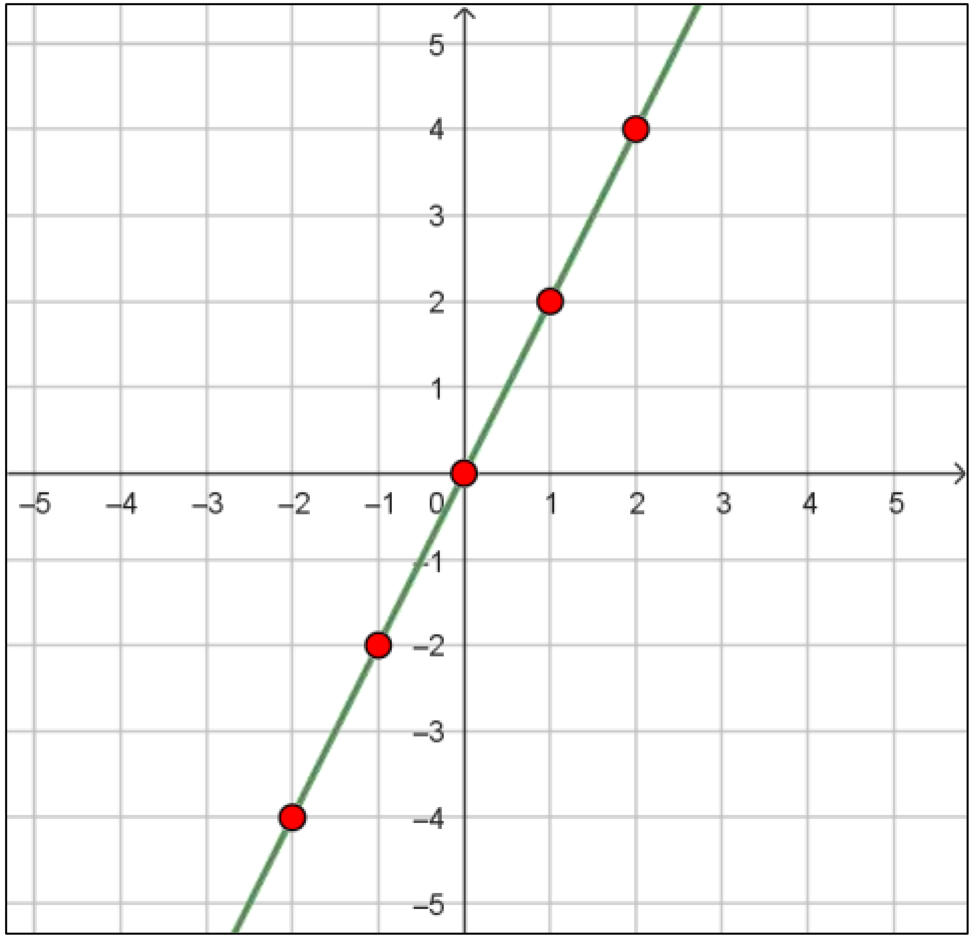
☞Line in the system of coordinates

In the system of coordinates, the line is a relation between the coordinates x and y. This relation is in the form of $y = ax + b$

Example 1:

$y = 2x$

x	-2	-1	0	1	2
y	-4	-2	0	2	4



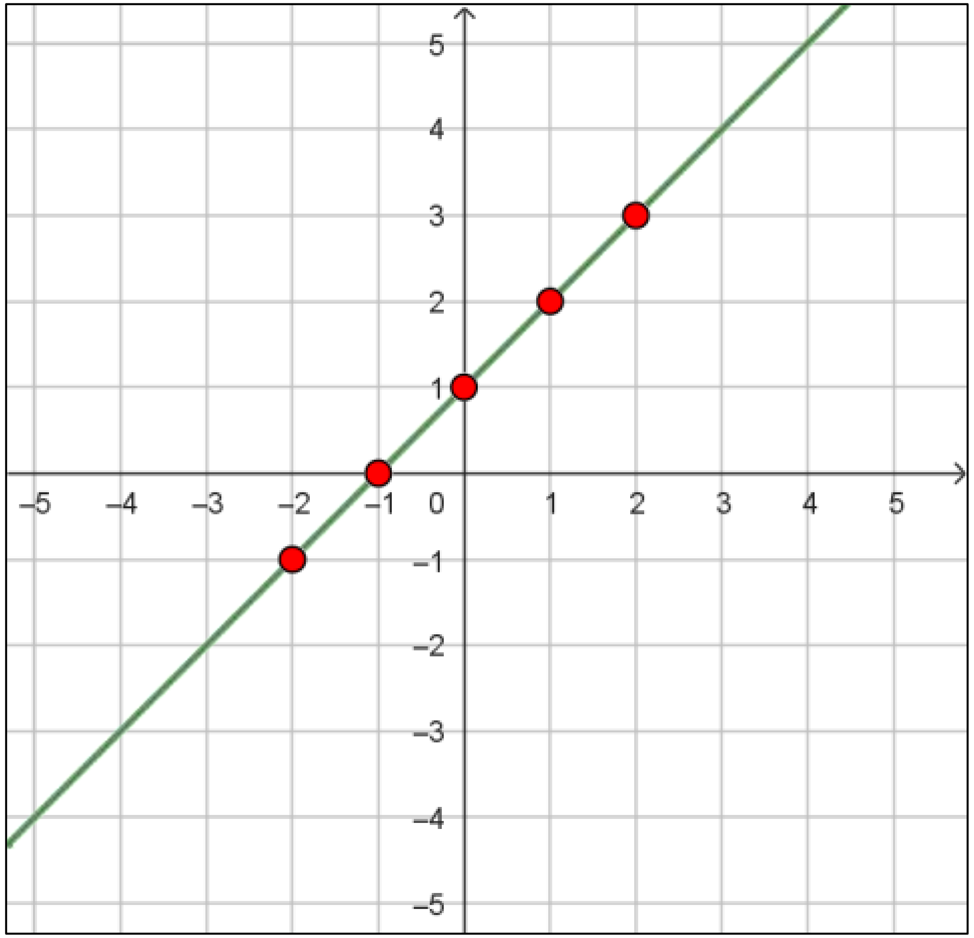
☞Line in the system of coordinates

In the system of coordinates, the line is a relation between the coordinates x and y. This relation is in the form of $y = ax + b$

Example 2:

$y = x + 1$

x	-2	-1	0	1	2
y	-1	0	1	2	3



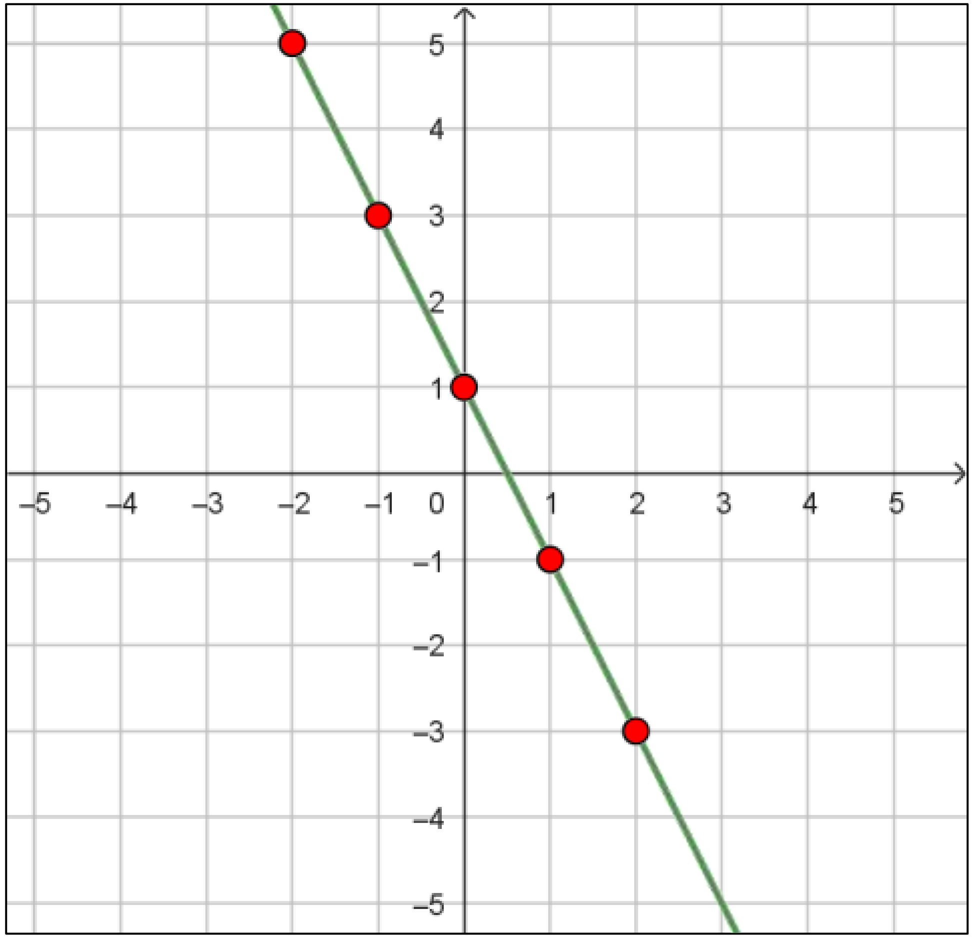
☞Line in the system of coordinates

In the system of coordinates, the line is a relation between the coordinates x and y. This relation is in the form of $y = ax + b$

Example 3:

$y = -2x + 1$

x	-2	-1	0	1	2
y	5	3	1	-1	-3



☞Line in the system of coordinates

$$y = ax + b$$

a → slope

b → y-intercept

☞Line in the system of coordinates

$$y = ax + b$$

- Determine the direction of a line.

How

Horizontal $a = 0$

Increasing $(a > 0)$

Decreasing $(a < 0)$

Vertical $(a \nexists)$

☞Line in the system of coordinates

$$y = ax + b$$

- The slope is the change in y coordinate with respect to the change in x coordinate of the line.

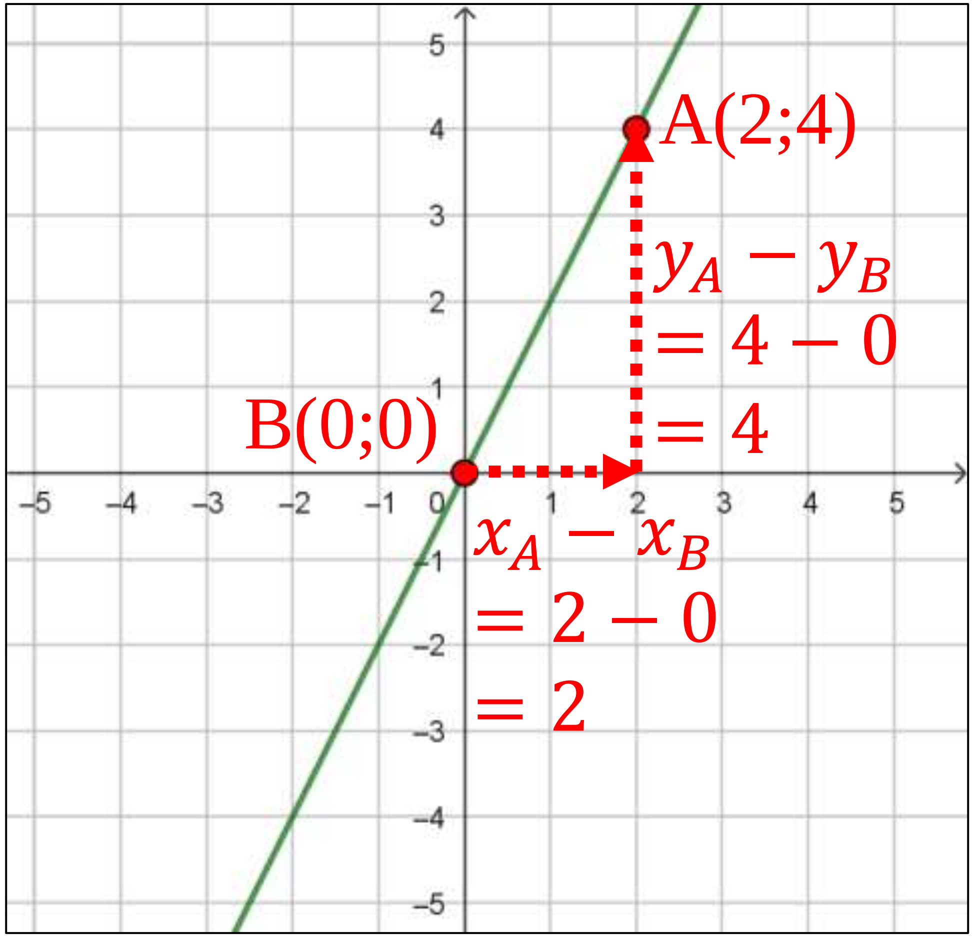
Slope:

$$a = \frac{y_A - y_B}{x_A - x_B} = \frac{4}{2} = 2$$

Example 1:

$y = 2x$

Slope is 2



☞Line in the system of coordinates

$$y = ax + b$$

- The slope is the change in y coordinate with respect to the change in x coordinate of the line.

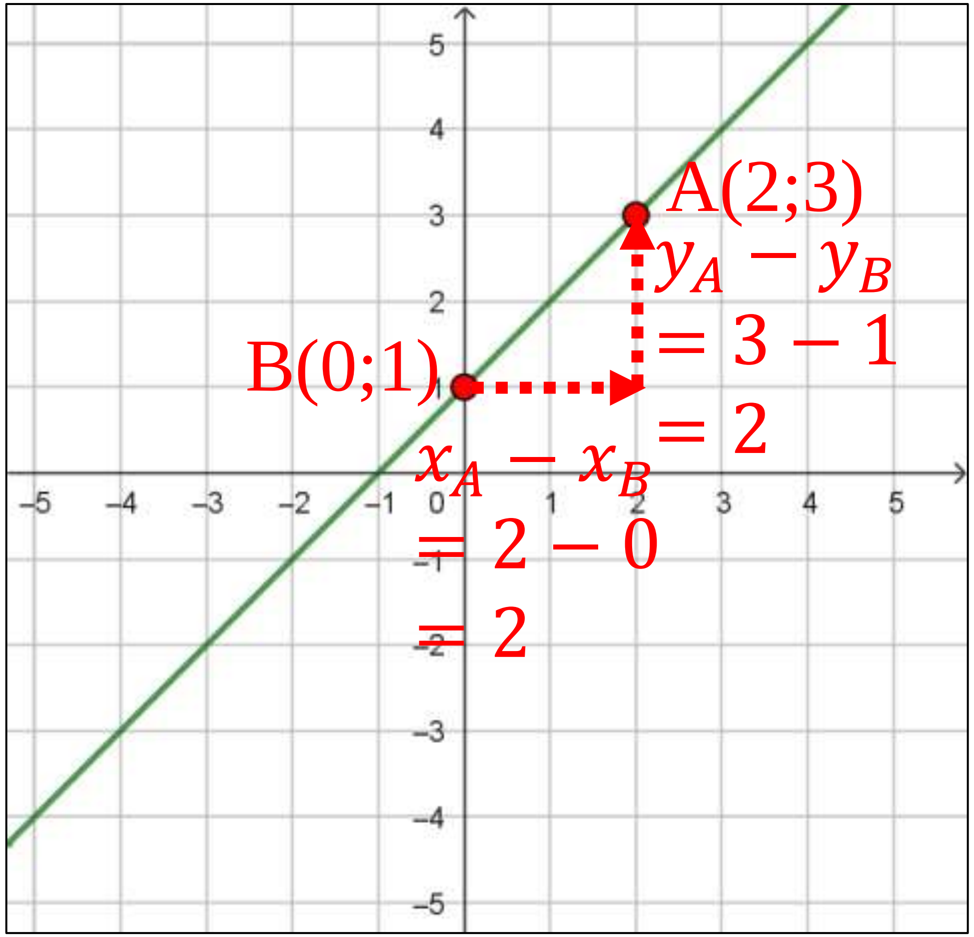
Slope:

$$a = \frac{y_A - y_B}{x_A - x_B} = \frac{2}{2} = 1$$

Example 2:

$y = x + 1$

Slope is 1



☞Line in the system of coordinates

$$y = ax + b$$

- The slope is the change in y coordinate with respect to the change in x coordinate of the line.

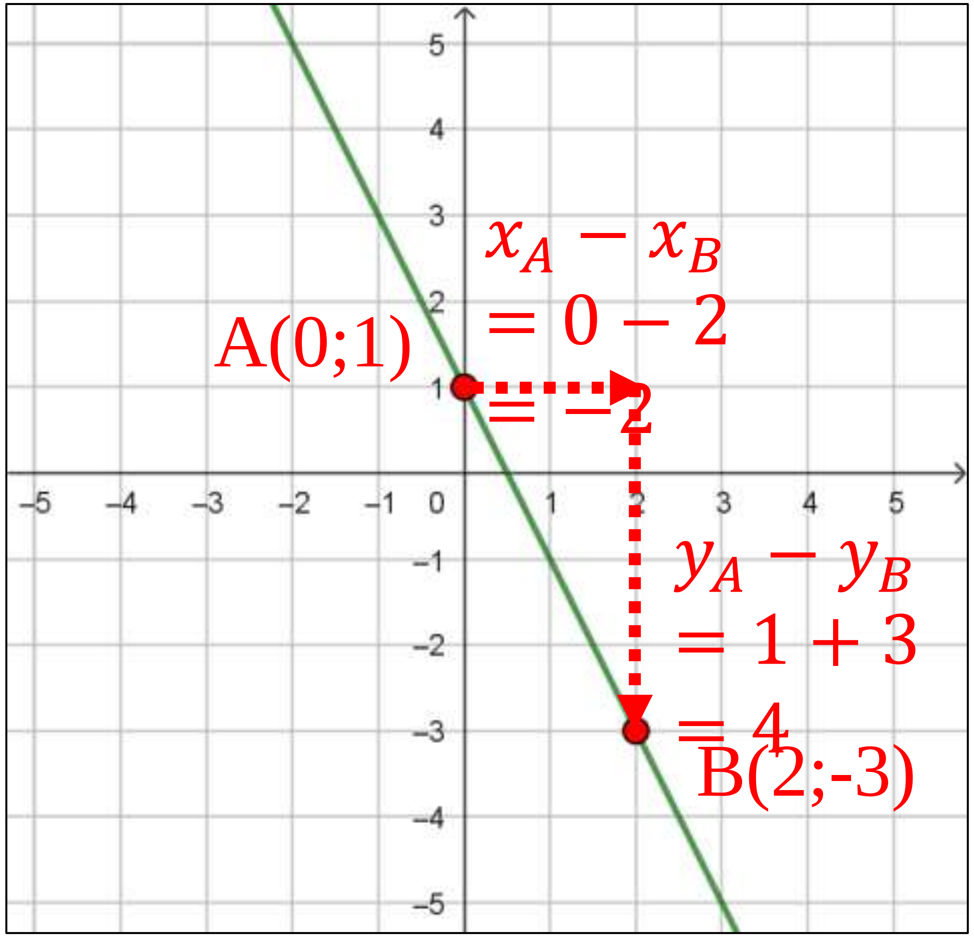
Slope:

$$a = \frac{y_A - y_B}{x_A - x_B} = \frac{4}{-2} = -2$$

Example 3:

$y = -2x + 1$

Slope is -2

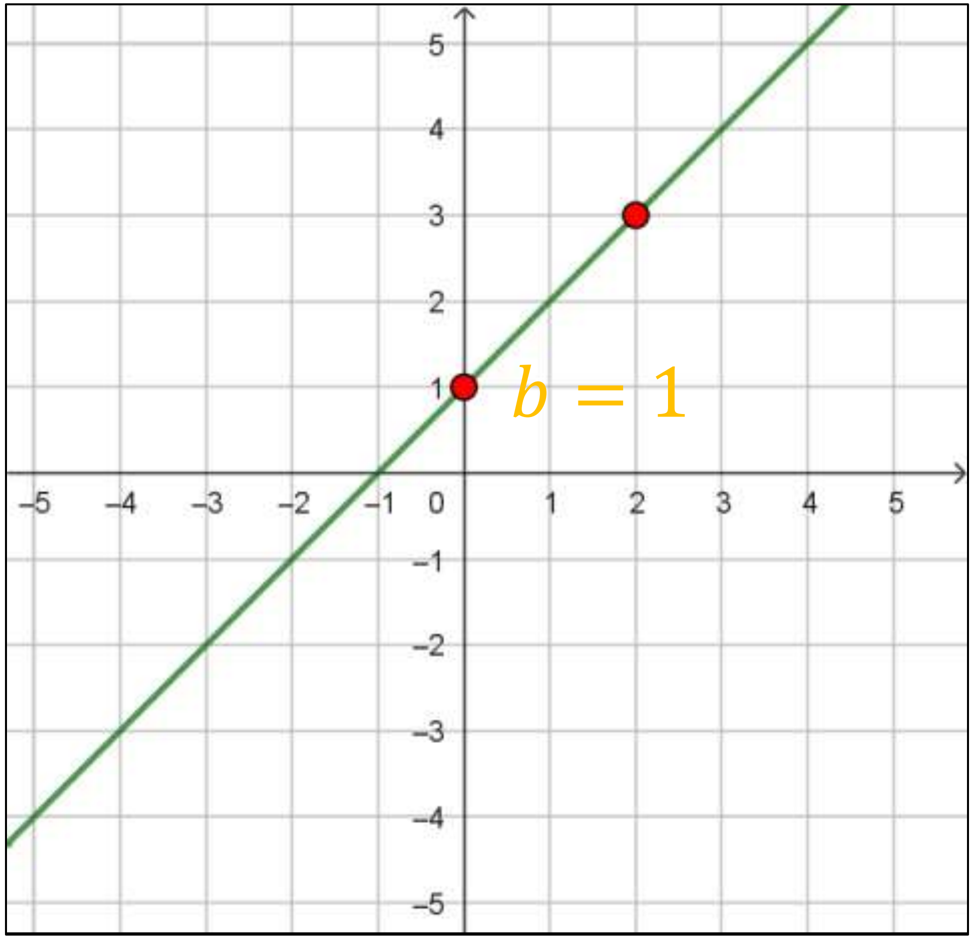


☞Line in the system of coordinates

$$y = ax \quad b$$

y-intercept

Intersection with y axis



☞ Application # 2

Determine the slope and the y-intercept in each case.

❶ $y = \frac{1}{2}x - 3$

Form of $y = ax + b$

So:

Slope $a = \frac{1}{2}$

y-intercept: $b = -3$

☞ Application # 2

Determine the slope and the y-intercept in each case.

❷ $y = -3x + 5$

Form of $y = ax + b$

So:

Slope $a = -3$

y-intercept: $b = 5$

☞ Application # 2

Determine the slope and the y-intercept in each case.

❸ $y = 3$

Form of $y = ax + b$

So:

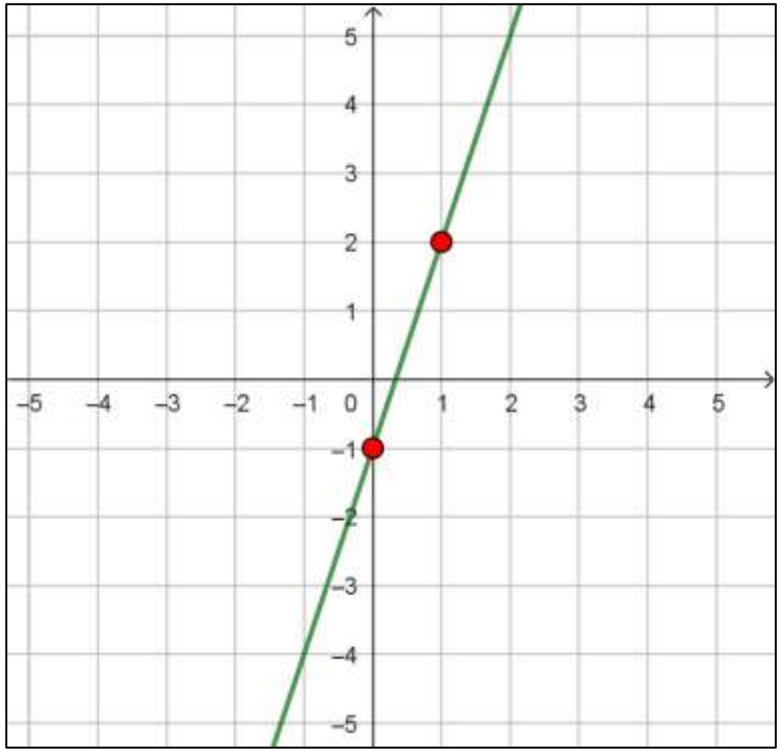
Slope $a = 0$

y-intercept: $b = 3$

☞ Application # 2

Determine the slope and the y-intercept in each case.

4



Suppose that $A(0;-1)$ and $B(1;2)$

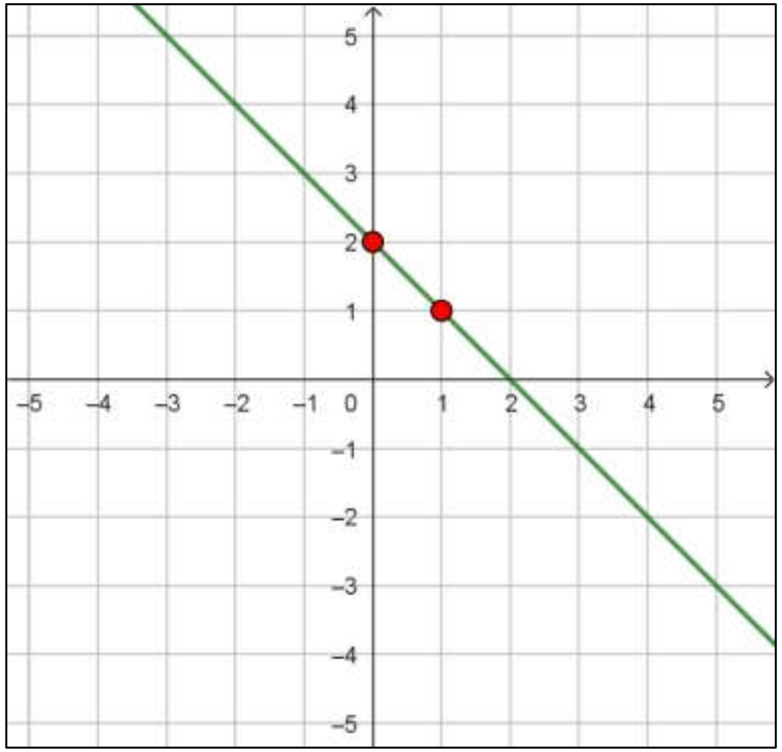
Slope: $a = \frac{y_A - y_B}{x_A - x_B} = \frac{-1-2}{0-1} = -\frac{3}{-1} = 3$

y-intercept: $b = -1$

☞ Application # 2

Determine the slope and the y-intercept in each case.

5



Suppose that $A(0;2)$ and $B(1;1)$

Slope: $a = \frac{y_A - y_B}{x_A - x_B} = \frac{2-1}{0-1} = \frac{1}{-1} = -1$

y-intercept: $b = 2$

☞Line in the system of coordinates

How to determine if a point belongs to a line?

Consider the line (d): $y = ax + b$ and the point $A(x_A; y_A)$.

We say,

A belongs to (d)

or

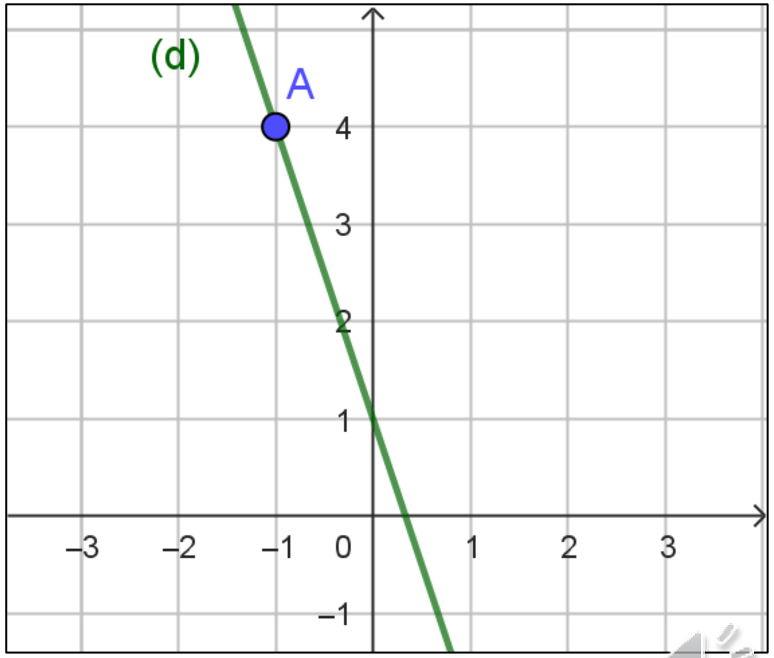
A is on (d)

or

(d) Passes through A

The coordinates of A verify the equation of (d)

$$y_A = ax_A + b$$



Example:

(d): $y = -3x + 1$; $A(-1; 4)$

$-3x_A + 1 = -3(-1) + 1 = 3 + 1 = 4 = y_A$

So A belongs to (d)

☞ Application # 3

Consider the line (d) of equation $y = -2x + 1$ and the two points A(3;-5) and B(2;3).

❶ Show that (d) passes through A and not through B.

❷ Calculate the slope of the line (AB).

❶ $-2x_A + 1 = -2(3) + 1 = -6 + 1 = -5 = y_A$

So (d) passes through A.

$-2x_B + 1 = -2(2) + 1 = -4 + 1 = -3 \neq y_A$

So (d) doesn’t pass through B.

☞ Application # 3

Consider the line (d) of equation $y = -2x + 1$ and the two points A(3;-5) and B(2;3).

❶ Show that (d) passes through A and not through B.

❷ Calculate the slope of the line (AB).

❷ $a_{(AB)} = \frac{y_B - y_A}{x_B - x_A} = \frac{3-(-5)}{2-3} = \frac{8}{-1} = -8$

☞Line in the system of coordinates

How to determine if two lines are parallel or perpendicular?

Consider the two lines (d): $y = ax + b$ and the line (d'): $y = a'x + b'$.

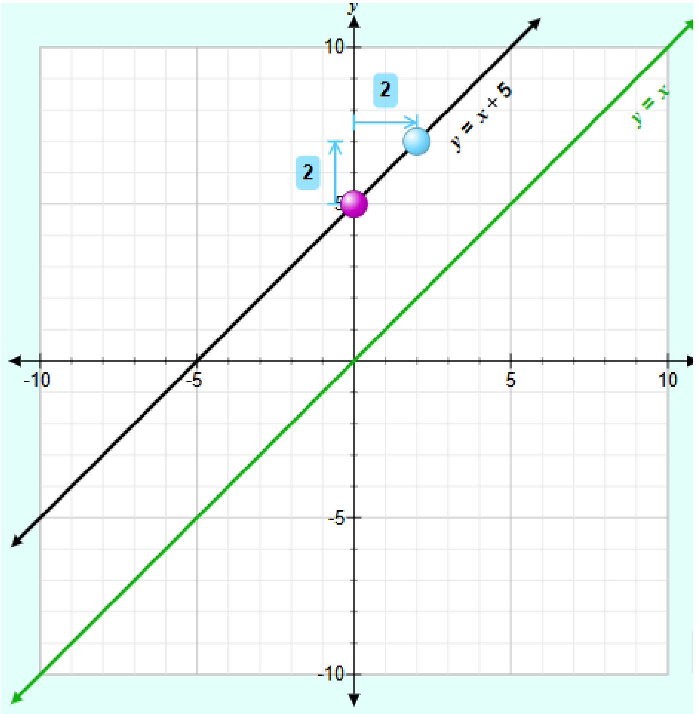
$a = a'$

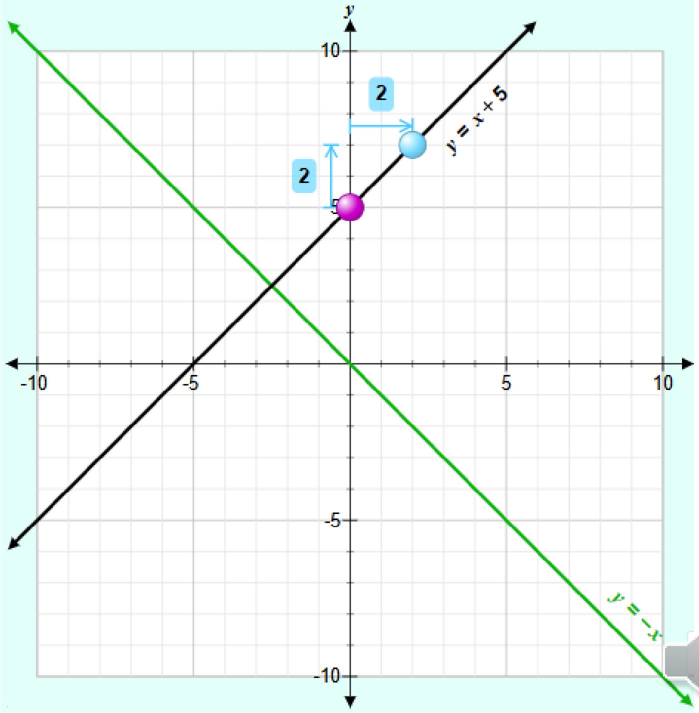
$b = b'$ → (d) = (d')

$b \neq b'$ → (d) // (d')

$a \times a' = -1$

(d) ⊥ (d')





☞ Application # 4

Study the relative position of the two lines in each case:

① (d): $y = 2x$ and (d'): $y = 2x + 1$

② (d): $y = 2x$ and (d'): $y = -\frac{1}{2}x + 1$

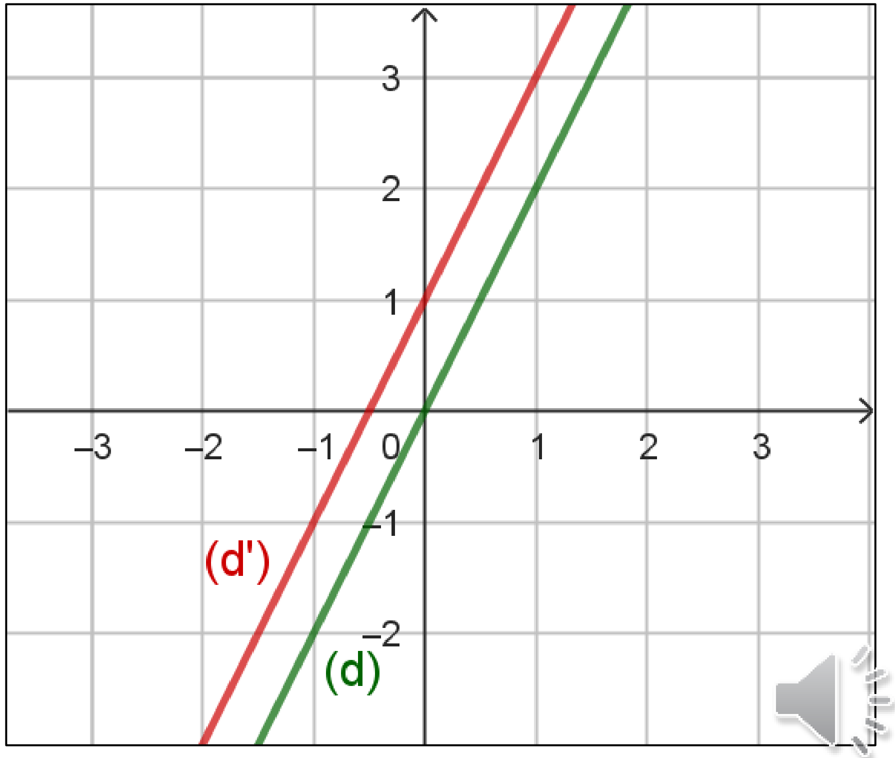
③ (d): $y = 2x - 3$ and (d'): $\frac{1}{2}y - x = -3$

④ (d): $y = -x + 3$ and (d'): $y = 2x + 1$

① $a = 2$; $b = 0$

$a' = 2$; $b' = 1$

$a = a'$ & $b \neq b'$ so (d)//(d')



☞ Application # 4

Study the relative position of the two lines in each case:

❶ (d): $y = 2x$ and (d'): $y = 2x + 1$

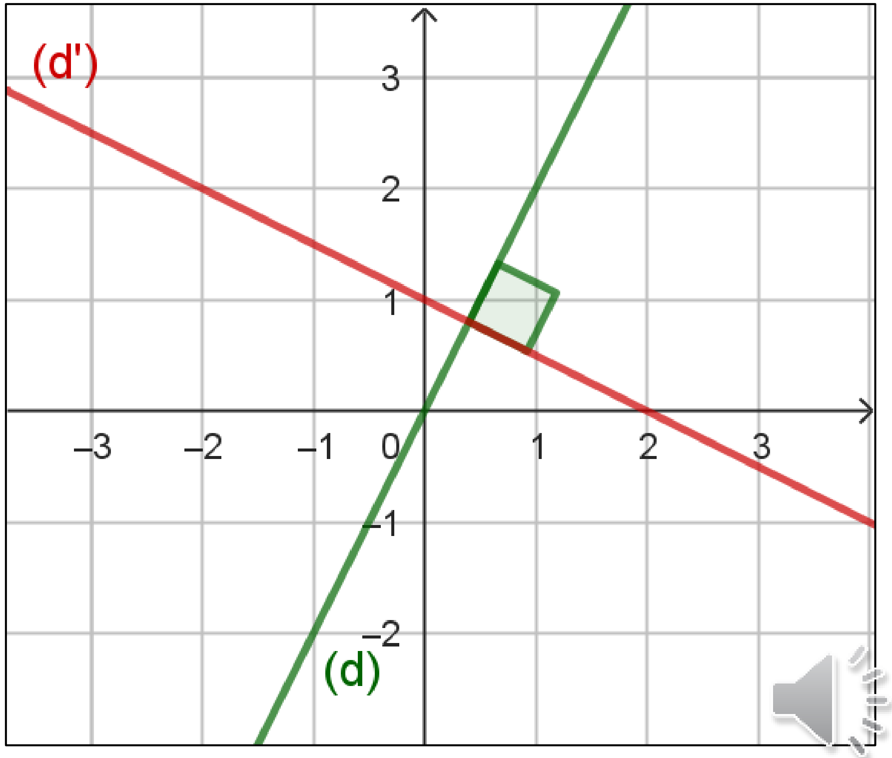
❷ (d): $y = 2x$ and (d'): $y = -\frac{1}{2}x + 1$

❸ (d): $y = 2x - 3$ and (d'): $\frac{1}{2}y - x = -3$

❹ (d): $y = -x + 3$ and (d'): $y = 2x + 1$

❷ $a \times a' = 2 \times \frac{-1}{2} = -1$

so (d) and (d') are perpendicular.



☞ Application # 4

Study the relative position of the two lines in each case:

1. (d): $y = 2x$ and (d'): $y = 2x + 1$
2. (d): $y = 2x$ and (d'): $y = -\frac{1}{2}x + 1$
3. (d): $y = 2x - 6$ and (d'): $\frac{1}{2}y - x = -3$
4. (d): $y = -x + 3$ and (d'): $y = 2x + 1$

3 (d'): $\frac{1}{2}y - x = -3$

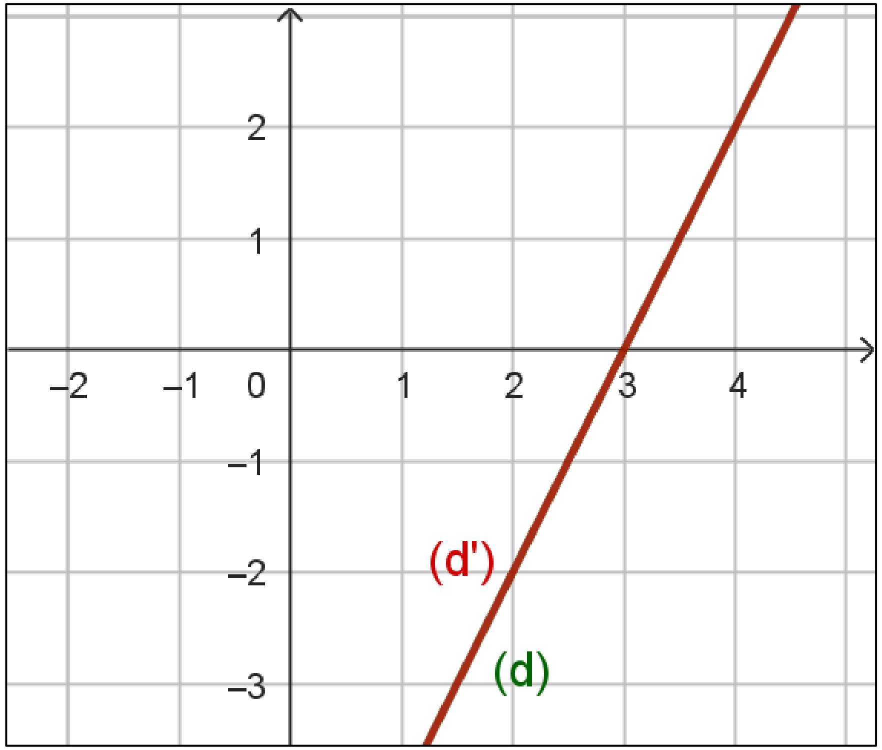
$$\frac{1}{2}y = x - 3$$

$$y = 2x - 6$$

$a = 2 \quad ; \quad b = -6$

$a' = 2 \; ; \; b' = -6$

$a = a'$ & $b = b'$ so (d) and (d') are confounded lines.



☞ Application # 4

Study the relative position of the two lines in each case:

① (d): $y = 2x$ and (d’): $y = 2x + 1$

② (d): $y = 2x$ and (d’): $y = -\frac{1}{2}x + 1$

③ (d): $y = 2x - 6$ and (d’): $\frac{1}{2}y - x = -3$

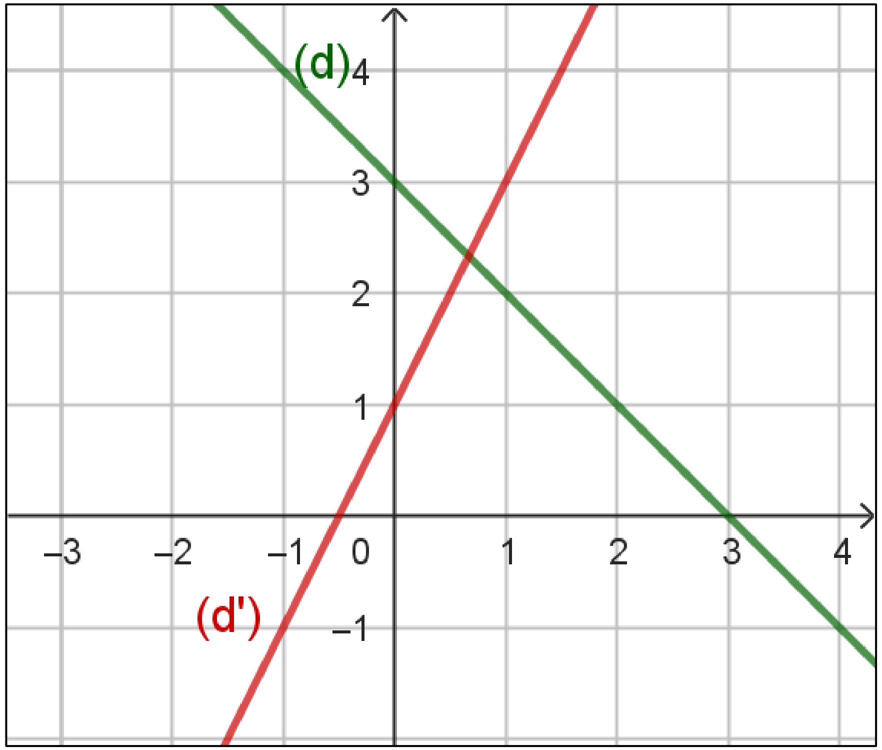
④ (d): $y = -x + 3$ and (d’): $y = 2x + 1$

④ $a = -1$; $a' = 2$

$a \times a' = -1 \times 2 = -2$

So (d) and (d’) are not perpendicular.

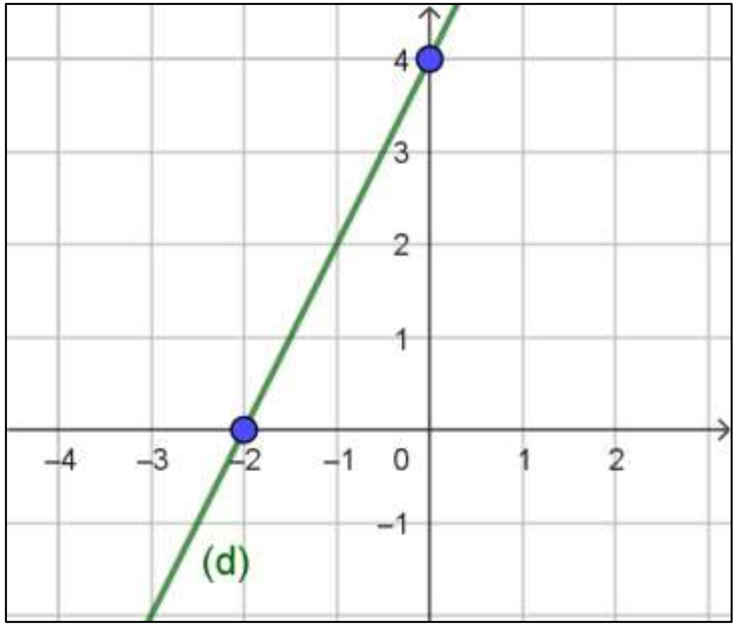
$a \neq a'$ so (d) and (d’) are not parallel.



☞Line in the system of coordinates

How to determine the x-intercept and the y-intercept if the equation is given?

Consider the two lines (d): $y = ax + b$.



x-intercept: $y = 0$

y-intercept: $x = 0$

Example: (d): $y = 2x + 4$

x –intercept: for $y = 0$; $0 = 2x + 4$; $2x = -4$; $x = -\frac{4}{2} = -2$

so (-2;0)

y –intercept: for $x = 0$; $y = 2(0) + 4 = 4$

so (0;4)

☞ Application # 5

Consider the line (d): $y = \frac{1}{2}x + 1$ and the point A(2;-2).

1. Show that (d) doesn't pass through A.
2. (d) intersect ($x'x$) at E and ($y'y$) at F. Find the coordinates of E and F.
3. Calculate the slope of the line (EA).
4. Does (EA) perpendicular to (d)? Justify.

❶ $\frac{1}{2}x_A + 1 = \frac{1}{2}(2) + 1 = 1 + 1 = 2 \neq y_A$

So (d) doesn't pass through A.

☞ Application # 5

Consider the line (d): $y = \frac{1}{2}x + 1$ and the point A(2;-2).

1. Show that (d) doesn't pass through A.
2. (d) intersect ($x'x$) at E and ($y'y$) at F. Find the coordinates of E and F.
3. Calculate the slope of the line (EA).
4. Does (EA) perpendicular to (d)? Justify.

2

For $y = 0$; $0 = \frac{1}{2}x + 1$

$$\frac{1}{2}x = -1$$

$x = -2$ so E(-2;0)

For $x = 0$; $y = \frac{1}{2}(0) + 1 = 1$ so F(0;1)

☞ Application # 5

Consider the line (d): $y = \frac{1}{2}x + 1$ and the point A(2;-2).

1. Show that (d) doesn't pass through A.
2. (d) intersect $(x'x)$ at E and $(y'y)$ at F. Find the coordinates of E and F.
3. Calculate the slope of the line (EA).
4. Does (EA) perpendicular to (d)? Justify.

3

$$a_{(EA)} = \frac{y_E - y_A}{x_E - x_A} = \frac{0-(-2)}{-2-2} = \frac{2}{-4} = -\frac{1}{2}$$

☞ Application # 5

Consider the line (d): $y = \frac{1}{2}x + 1$ and the point A(2;-2).

1. Show that (d) doesn't pass through A.
2. (d) intersect $(x'x)$ at E and $(y'y)$ at F. Find the coordinates of E and F.
3. Calculate the slope of the line (EA).
4. Does (EA) perpendicular to (d)? Justify.

4

$$a_{(EA)} = -\frac{1}{2}$$

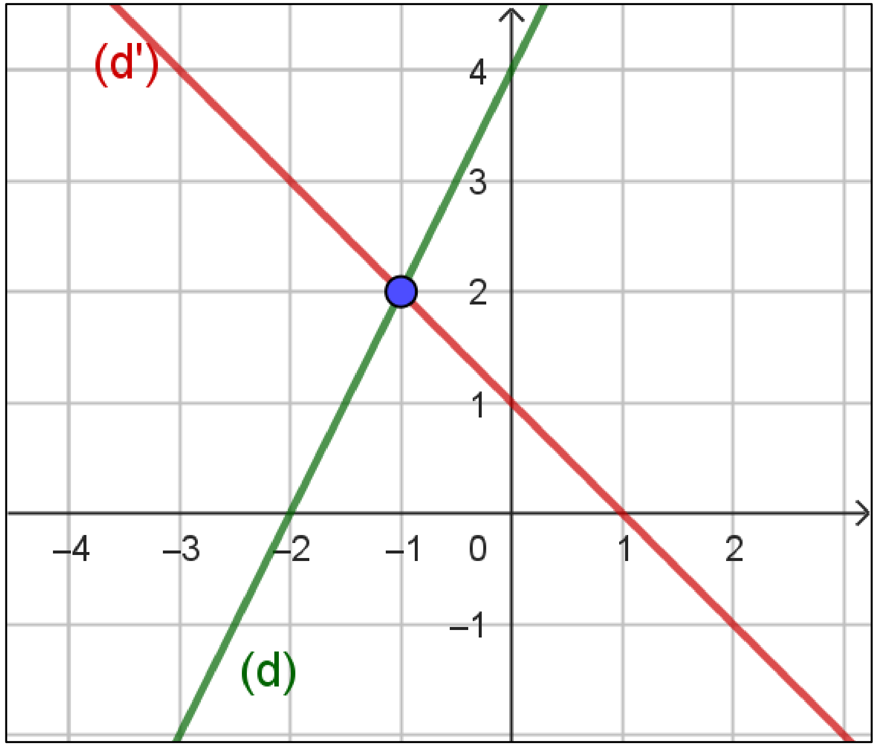
$$a_{(d)} \times a_{(EA)} = \frac{1}{2} \times \frac{-1}{2} = -\frac{1}{4} \neq -1$$

☞Line in the system of coordinates

How to determine the intersection point of two lines?

Suppose that the intersection point is called I.

(d): $y = ax + b$ (d’): $y = a'x + b'$



I belongs to (d) $y_I = ax_I + b$

and

I belongs to (d’) $y_I = a'x_I + b'$

$y_I = y_I$

Example: (d): $y = 2x + 4$; (d’): $y = -x + 1$

I belongs to (d): $y_I = 2x_I + 4$

I belongs to (d’): $y_I = -x_I + 1$

$y_I = y_I$

$2x_I + 4 = -x_I + 1$

$2x_I + x_I = 1 - 4$

$3x_I = -3$

$x_I = -\frac{3}{3} = -1$

$y_I = -x_I + 1 = -(-1) + 1 = 2$

So I(-1;2)